



**12 MATHEMATICS METHODS**  
**COMMON TEST 3 – Term 1 2016**  
*Integration Techniques*

Name: \_\_\_\_\_

Marks: \_\_\_\_\_ / 43

**Instructions:**

- External notes are not allowed
- Duration of test: 40 minutes
- This test contributes to 6% of the year (school) mark
- No calculator

Full marks may not be awarded to correct answers unless sufficient justification is given.

---

**1. (3 marks)**

Evaluate  $\int_{-\pi}^{\pi} \cos(x/2) dx$

**2. (8 marks)**

Determine the following integrals:

(a)  $\int \frac{3}{x^{-2}} + 4 \, dx$  [2]

(b)  $\int \frac{(4-x)}{\sqrt{x}} \, dx$  [3]

(c)  $\int \frac{1}{(2x-1)^5} \, dx$  [3]

3. (7 marks)

Evaluate

(a)  $\frac{d}{dx} \int_{-4}^x \sqrt{5t^2 - 3} dt$

[1]

(b)  $\frac{d}{dx} \int_{-1}^{-x^3} \frac{t}{(t-2)^2} dt$

[3]

(c)  $\int_{2x}^1 \frac{d}{dt} [t\sqrt{1+t^2}] dt$

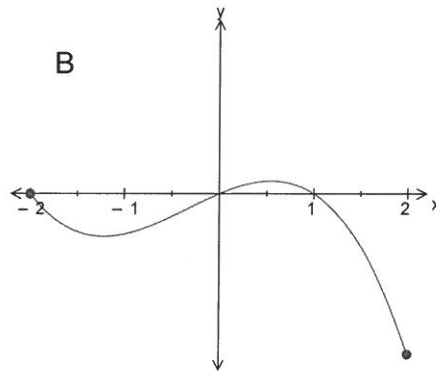
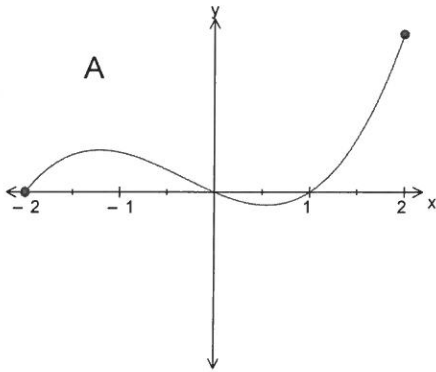
[3]

4. (5 marks)

Two functions  $f(x)$  and  $g(x)$  exist such that:

$$\int_{-2}^0 f(x) dx = 2 \quad \text{and} \quad \int_1^0 g(x) dx = -1$$

(a) Determine which of the following graphs are  $f(x)$  and  $g(x)$ . [2]



(b) Answer true or false for each of the following. [3]

(i)  $\int_{-2}^2 f(x) dx > \int_{-2}^2 g(x) dx$

(ii)  $\int_0^2 f(x) dx > \int_{-2}^2 f(x) dx$

(iii)  $\int_{-2}^2 g(x) dx > 0$

5. (3 marks)

The gradient function of a curve is given by  $\frac{dy}{dx} = x^2 - 4e^{-2x}$

Find the equation of this curve given it passes through the point (0 , 3)

**6. (7 marks)**

(a) Find an approximation to the area of the region between

$y = e^{2x}$ , and the lines  $x = 0$ ,  $x = 3$  and the  $x$  axis using exact values and

(i) 3 left rectangles [2]

(ii) 3 right rectangles [1]

(iii) The average of parts (i) and (ii) [1]

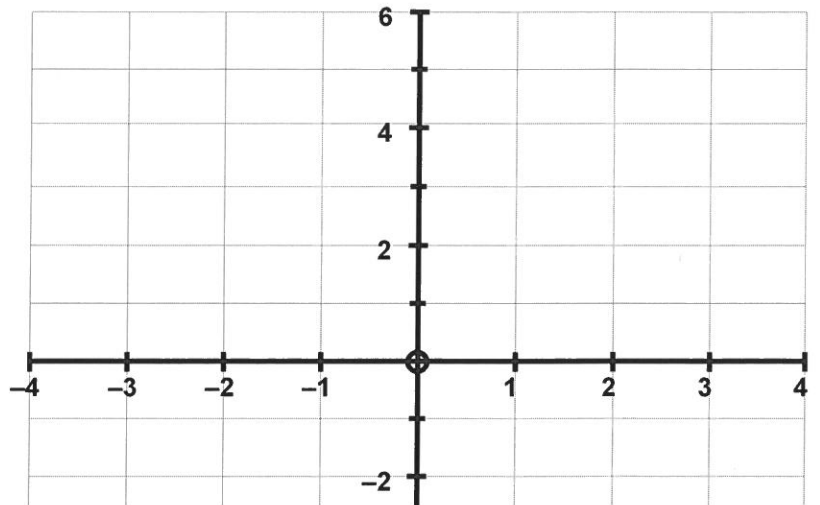
(b) Evaluate using exact values  $\int_0^3 e^{2x} dx$  [2]

(c) Explain why the answers in parts (a) and (b) are different. [1]

**7 (5 marks)**

Consider  $A(x) = \int_{-1}^x (-t + 1) dt$

Plot  $f(t) = -t + 1$



(a) Find

$A(-1)$

$A(0)$

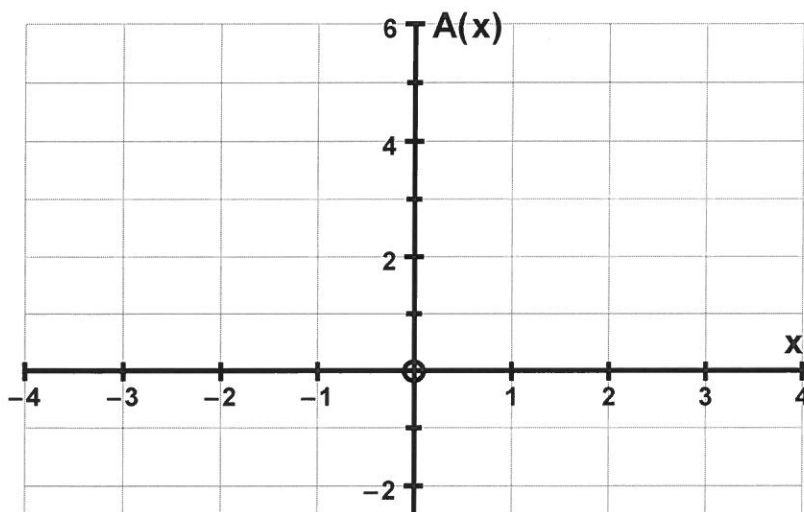
$A(1)$

$A(2)$

[2]

(b) Plot the values in part (a) and hence sketch the graph of  $A(x)$  for  $-1 \leq x \leq 2$

[1]



(c) Determine the defining rule for (i)  $A'(x)$

(ii)  $A(x)$

[2]

8. (5 marks)

(a) Find  $\frac{dy}{dx}$  given that  $y = x \cos x$  [2]

(b) Use your answer in part (a) to find  $\int (x \sin x) dx$  [3]